

MIDTERM EXAMINATION - SOLUTIONS

November 8, 2004

Time Allowed: 2 hours

Professor: L. D. Wegner

Marks

- 6 1. What is the difference between a crystal structure, a crystal system, and a unit cell?

A crystal system refers to the shape of the parallelepiped used to describe the arrangement of atoms.

A crystal structure adds the position of atoms within the crystal system.

A unit cell is a representative volume or basic structural unit of the crystal structure, many of which may be stacked together to form the complete three-dimensional lattice. It includes the parallelepiped and the portions of all atoms within it.

- 6 2. List two methods used to strengthen a metal and briefly explain the mechanism by which each works to strengthen the material.

These may be found on page 96 of the notes. The basic mechanism of strengthening for all the methods is to provide barriers to the motion of dislocations. How this is achieved differs for each method. As an example, strain hardening involves deforming a metal plastically. As plastic deformation takes place, more dislocations are formed, resulting in more barriers to the motion of other dislocations, thereby increasing the strength of the metal.

- 4 3. Why does a brittle material typically fracture prior to reaching its yield strength?

Brittle materials contain flaws which produce stress concentrations. These stress concentrations lead to very high stresses locally, causing failure initiated at these locations at overall stress levels that are very low relative to the material's yield strength.

- 2 4. What are the driving forces for the creation of atomic bonds? In other words, why do atoms bond with each other?

1. The desire for atoms to achieve stable electron configurations.

2. Minimization of energy.

- 4 5. Why do grain boundaries exist in many materials?

For crystalline materials, as the material cools from the molten state, crystals begin to nucleate at many places at the same time. The crystal structures which grow around these nucleation points all form at different orientations. When the individual growing crystal grains encounter each other, grain boundaries are formed.

- 6 6. What do the variables Q and T represent in *Arrhenius equation*? Explain why processes controlled by the *Arrhenius equation* are dependent on these variables.

Q = activation energy

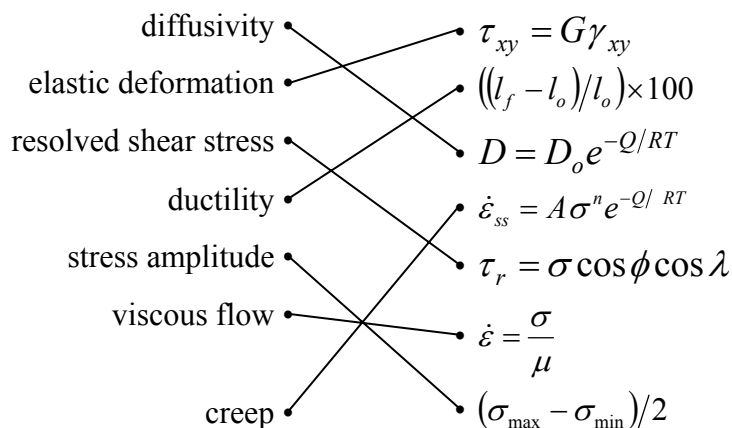
T = absolute temperature

The Arrhenius equation represents processes that involve the need for individual atoms to overcome an energy barrier in order for the process to proceed. This energy barrier is the activation energy, Q . As the temperature increases, a larger percentage of atoms possess the required activation energy, and thus the process accelerates as the temperature increases.

- 6 7. At room temperature, a certain polystyrene exhibits a Young's modulus of approximately 3.0 MPa, while at a temperature of 150°C, Young's modulus is only 0.4 MPa. What is happening at the molecular scale that results in this reduction?

Polystyrene is a polymer which consists of long chainlike molecules. At colder temperatures, individual molecule chains are connected by a large number of secondary bonds. As the temperature increases above the glass transition temperature, the secondary bonds "melt" or disappear. The presence of the bonds at lower temperatures makes the material more resistant to stretching, exhibited by a higher Young's modulus. At temperatures above the glass transition temperature, the absence of the secondary bonds makes it easier for the chains to move relative to each other, resulting in a more flexible material with a lower Young's modulus.

- 6 8. Match the concept on the left with the appropriate equation on the right by drawing a line between the two.



- 10 9. Using the information listed in the supplemental material on the last page of this exam, compute the theoretical density of iron.

$$\rho = \frac{nA}{V_c N_A}$$

$$\text{For BCC: } n = 2, V_c = a^3, a = \frac{4r}{\sqrt{3}} \rightarrow V_c = \frac{64r^3}{3\sqrt{3}}$$

$$\text{For iron: } r = 0.124 \times 10^{-7} \text{ cm} \rightarrow V_c = \frac{64(0.124 \times 10^{-7})^3}{3\sqrt{3}} = 2.348 \times 10^{-23} \text{ cm}^3$$

$$A = 55.85 \text{ g/mol}$$

$$\rho = \frac{2 \cdot 55.85 \text{ g/mol}}{(2.348 \times 10^{-23} \text{ cm}^3)(6.023 \times 10^{23} \text{ /mol})} = 7.90 \text{ g/cm}^3$$

- 10 10. The number of vacancies present in silver at 900°C is $10^{24}/\text{m}^3$. Calculate the number of vacancies at 500°C given that the energy for vacancy formation is $1.06 \times 10^5 \text{ J/mol}$; assume that the density at both temperatures is the same.

$$N_v = Ne^{-Q_v/RT}$$

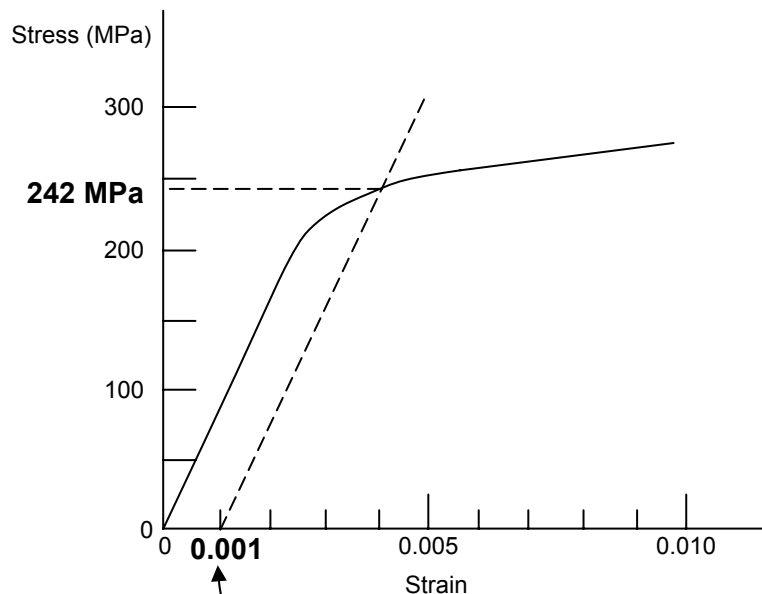
At 900°C : $T = 1173\text{K}$, $N_v = 10^{24}/\text{m}^3$, $Q_v = 1.06 \times 10^5 \text{ J/mol}$

$$10^{24} = Ne^{-1.06 \times 10^5 \text{ J/mol} / (8.31 \text{ J/mol}\cdot\text{K})(1173 \text{ K})} \rightarrow N = 5.28 \times 10^{28} / \text{m}^3$$

At 500°C : $T = 773\text{K}$, $N = 5.28 \times 10^{28} / \text{m}^3$, $Q_v = 1.06 \times 10^5 \text{ J/mol}$

$$N_v = 5.28 \times 10^{28} e^{-1.06 \times 10^5 \text{ J/mol} / (8.31 \text{ J/mol}\cdot\text{K})(773 \text{ K})} \rightarrow N_v = 3.60 \times 10^{21} \text{ vacancies}/\text{m}^3$$

- 10 11. Consider a brass alloy with the stress-strain behaviour shown in the figure below. A cylindrical specimen of this material 15.00 mm in diameter and 100.00 mm long is pulled in tension, after which the tensile load is released. After the load is released, the length of the specimen is 100.10 mm. Compute the magnitude of the force necessary to cause this elongation.



Plastic strain after the load has been released: $\epsilon_p = \frac{100.10 - 100.00}{100.00} = 0.001$

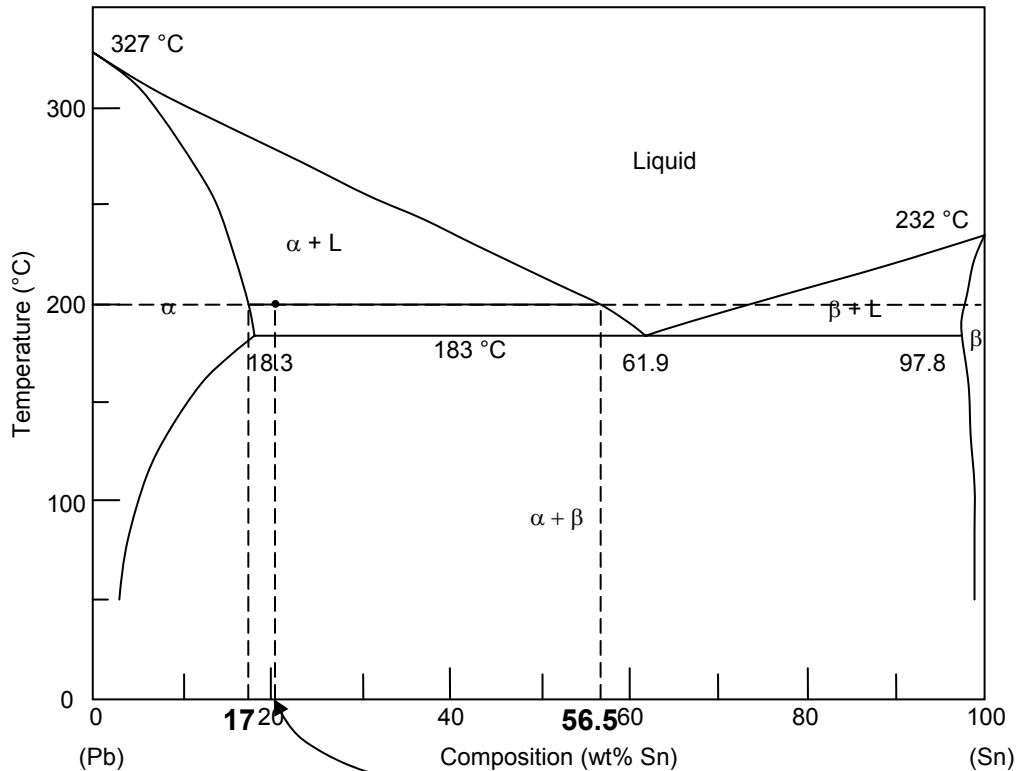
Stress to which the specimen must be subjected to in order to have this plastic strain (accounting for elastic strain recovery): $\sigma_{\max} = 242 \text{ MPa}$

Cross sectional area of specimen: $A = \frac{\pi}{4} (15.00 \text{ mm})^2 = 176.7 \text{ mm}^2$

Corresponding force: $F = \sigma_{\max} A = 242 \frac{\text{N}}{\text{mm}^2} \cdot 176.7 \text{ mm}^2 = 42.8 \text{ kN}$

10 12. A material system is composed of 23 g of tin (Sn) and 90 g of lead (Pb) and is held at a temperature of 200°C. Showing your work on the phase diagram below, answer the following questions:

- What are the phases present?
- What is the composition of each phase?
- What is the mass of each phase?



Overall composition: $C_o = \frac{23}{23 + 90} = 0.204$ or 20.4 wt% Sn

- (a) Phases present: α (solid solution of Sn in Pb)
L (liquid)

(b)

$$C_\alpha = 17 \text{ wt\% Sn}$$

$$C_L = 56.6 \text{ wt\% Sn}$$

(c) $W_L = \frac{20.4 - 17}{56.5 - 17} = 0.086 \quad \rightarrow \quad m_L = (0.086)(90 \text{ g} + 23 \text{ g}) = 9.7 \text{ g}$

$W_\alpha = \frac{56.5 - 20.4}{56.5 - 17} = 0.914 \quad \rightarrow \quad m_\alpha = (0.914)(90 \text{ g} + 23 \text{ g}) = 103.3 \text{ g}$

- 10 13. Carbon is allowed to diffuse through a steel plate 10 mm thick at 950K. The concentrations of carbon at the two faces are 0.85 and 0.40 kg/m³, which are maintained constant. If the pre-exponential and activation energy for the diffusion of carbon in steel are $6.2 \times 10^{-7} \text{ m}^2/\text{s}$ and 80,000 J/mol, respectively, compute the diffusion area required in order for the passage of $1.25 \times 10^{-6} \text{ kg}$ of carbon in 10,000 s.

$$J = -D \frac{dc}{dx} \quad D = D_0 e^{-Q/RT}$$

$$D_0 = 6.2 \times 10^{-7} \text{ m}^2/\text{s} \quad Q = 80,000 \text{ J/mol} \quad T = 950 \text{ K}$$

$$\frac{dc}{dx} = \frac{(0.40 - 0.85) \text{ kg/m}^3}{0.01 \text{ m}} = -45 \text{ kg/m}^2$$

$$D = (6.2 \times 10^{-7} \text{ m}^2/\text{s}) e^{\frac{-80000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(950 \text{ K})}} = 2.463 \times 10^{-11} \text{ m}^2/\text{s}$$

$$J = -(2.463 \times 10^{-11} \text{ m}^2/\text{s})(-45 \text{ kg/m}^2) = 1.108 \times 10^{-9} \text{ kg/m}^2\text{s}$$

$$\frac{1.25 \times 10^{-6} \text{ kg}}{10,000 \text{ s} \cdot A} = 1.108 \times 10^{-9} \text{ kg/m}^2\text{s} \quad \rightarrow \quad \boxed{A = 0.113 \text{ m}^2}$$

- 10 14. Calculate the fatigue crack propagation rate for a **surface crack** of length 2.0 mm when $\sigma_{\max} = +150 \text{ MPa}$ and $\sigma_{\min} = -50 \text{ MPa}$. The values for m and A are 3.0 and 1.8×10^{-7} , respectively, for stress in MPa and crack length in m, and which yields a crack propagation rate in mm/cycle. Assume that the dimensionless parameter Y has a value of 1.5.

We must solve for $\frac{da}{dN} = A(\Delta K)^m$ in which

$$A = 1.8 \times 10^{-7}$$

$$m = 3.0$$

$$\Delta K = Y \Delta \sigma \sqrt{\pi a}$$

$$a = 0.002 \text{ m}$$

$$Y = 1.5$$

$$\Delta \sigma = 150 \text{ MPa (compression doesn't count)}$$

$$\rightarrow \Delta K = (1.5)(150 \text{ MPa})\sqrt{\pi(0.002 \text{ m})} = 17.83 \text{ MPa}\sqrt{\text{m}}$$

$$\rightarrow \frac{da}{dN} = 1.8 \times 10^{-7} (17.83)^{3.0} = \boxed{0.0010 \text{ mm/cycle}} \text{ (notice that this equation}$$

requires ΔK in units of $\text{MPa}\sqrt{\text{m}}$ to give the crack propagation rate in mm/cycle)